An Analytical Solution for Blast Waves

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An approximate analytical method that is valid for the very low shock Mach number regime of a blast wave is described in the present paper. The method is based on the assumption of a power-law density profile behind the blast wave. The exponent of the power-law density profile is determined for each local shock Mach number from the mass integral. Solution of the differential equations of continuity and momentum then yield the particle velocity and the pressure distributions. The dependence of the shock decay coefficient on the shock velocity is determined from the energy integral. Comparison with the exact numerical solution of Goldstine and von Neumann and other existing analytical solutions indicates that the present solution is surprisingly accurate for the very low shock strength regime as compared to existing analytical solutions.

1.0 Introduction

THE theory of point blast waves is a very fundamental one in gasdynamics and has been applied to a variety of problems in hypersonic aerodynamics,[1,2] astrophysics,[3,4] and hypervelocity impact.[5,6] The validity of the classical self-similar solution of von Neumann,[7] Taylor,[8] and Sedov[9] is confined to the early time regime when the shock wave is very strong (i.e., $1/M_s^2 \to 0$). For intermediate times when the shock strength is finite, small departures from the classical self-similar solution due to counterpressure effects is accounted for in the perturbation solution of Sakurai[10] and the "quasi-similar" solution of Oshima.[11] For the asymptotic motion at late times when $1/M_s^2 \to 1$, solutions have been obtained by Whitham[12] and Sedov.[9] Apart from exact numerical solutions,[13] there appears to be a lack of an accurate analytical solution that provides an adequate description of the blast motion for the entire regime. In the present paper we describe an approximate analytical method that gave a surprisingly accurate solution for the entire propagation regime of a blast wave.

The present method is due to Rae[6] who laid down most of the essential steps of the analyses but did not carry out the work to completion. Following Rae, the analysis was further developed by Lee.[14] The essence of the method is to assume a power-law density profile behind the blast wave, the exponent of which is determined from the mass integral. This then enables the particle velocity profile to be obtained from the differential equation of mass conservation. With the form for the density and particle velocity profiles known, the momentum equation can be integrated directly to determine the pressure profile. Substituting these profiles into the energy integral then yields a first-order differential equation for the dependence of the decay coefficient on the shock Mach number. The integration of this equation with the given boundary conditions then completes the solution of the problem. It should be noted that the assumption of a power-law density profile was first proposed by Porzel,[15] and it was Rae[6] who pointed out the use of the energy integral for the solution of the problem.

At the completion of the present work, it was found that a similar attempt has been made by Sakurai[16] to obtain an analytical solution valid for the entire propagation regime of the blast wave. In Sakurai's work, a linear velocity profile was first assumed. To obtain the density profile, it was necessary to assume further that the derivative of the density with respect to the shock Mach number in the continuity equation can be neglected. This assumption is true only for relatively high shock Mach number and considerable errors are involved in the asymptotic regime when $1/M_s^2 \to 1$. The rest of Sakurai's analysis is similar to the present work. A more detailed discussion of Sakurai's analysis and the accuracy of his results will be given in later sections of this paper.

2.0 Theoretical Analysis

Following Rae,[6] the basic conservation equations for the unsteady one-dimensional adiabatic motion of the perfect gas behind the expanding blast wave can be written as

Conservation of mass:

$$(\phi - \xi)\partial\psi/\partial\xi + \psi(\partial\phi/\partial\xi) + j\phi(\psi/\xi) = 2\theta\eta(\partial\psi/\partial\eta) \quad (1)$$

Conservation of momentum:

$$(\phi - \xi)\partial\phi/\partial\xi + \theta\phi + (1/\psi)\partial f/\partial\xi = 2\theta\eta(\partial\phi/\partial\eta) \quad (2)$$

Conservation of energy:

$$(\phi - \xi)\left(\frac{\partial f}{\partial\xi} - \frac{\gamma f}{\psi}\frac{\partial\psi}{\partial\xi}\right) + 2\theta f = 2\theta\eta\left(\frac{\partial f}{\partial\eta} - \frac{\gamma f}{\psi}\frac{\partial\psi}{\partial\eta}\right) \quad (3)$$

where the dimensionless parameters are defined as follows:

$$\phi(\xi,\eta) = u(r,t)/\dot{R}_s(t)$$

$$f(\xi,\eta) = p(r,t)/(\rho_0\dot{R}_s^2)$$

$$\psi(\xi,\eta) = \rho(r,t)/\rho_0$$

$$\theta(\eta) = R_s\ddot{R}_s/\dot{R}_s^2,\ \xi = r/R_s(t),\ \eta = c_0^2/\dot{R}_s^2 = 1/M_s^2$$

and the numerical constant in Eq. (1) $j = 0, 1, 2$ for planar, cylindrical, and spherical waves. Conserving the total mass and the total energy enclosed by the blast wave at any instant of time, one gets the mass and energy integrals as follows:

Mass integral

$$\int_0^1 \psi\xi^j d\xi = \frac{1}{j+1} \quad (4)$$

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Energy integral:

$$1 = y\left(\frac{I}{\eta} - \frac{1}{\gamma(\gamma - 1)(j + 1)}\right) \quad (5)$$

where

$$I = \int_0^1 \left(\frac{f}{\gamma - 1} + \frac{\psi\phi^2}{2}\right)\xi^j d\xi \quad (6)$$
$$y = (Rs/R_0)^{j+1}$$

$R_0 = (E_0/\rho_0 c_0{}^2 k_j)^{1/(j+1)}$ (characteristic explosion length)

and $k_j = 1, 2\pi, 4\pi$ for $j = 0, 1, 2$.

The boundary conditions at the shock front $\xi = 1$ are the standard normal shock relationships given as

$$\phi(1,\eta) = [2/(\gamma + 1)](1 - \eta) \quad (7)$$

$$f(1,\eta) = [2/(\gamma + 1)] - [(\gamma - 1)/\gamma(\gamma + 1)]\eta \quad (8)$$

$$\psi(1,\eta) = (\gamma + 1)/(\gamma - 1 + 2\eta) \quad (9)$$

A physical boundary condition that the solution must satisfy is that $\phi(0,\eta) = 0$, which simply implies that the particle velocity is zero at the centre of symmetry. The mathematical formulation is now complete and the solution of Eqs. (1–3) satisfying the boundary conditions at the front [i.e., Eqs. (7) and (8)] and the energy integral [i.e., Eq. (5)] then defines the dynamics of the blast wave and its flow structure.

2.1 Density, Particle Velocity, and Pressure Profiles

On examining the basic conservation equations, it can be seen that the continuity equation [i.e. Eq. (1)] contains only two dependent variables, the density ψ and the particle velocity ϕ. Hence by assuming the form of the solution for one of them, the other can be found by solving the continuity equation. In Sakurai's[16] analysis, the particle velocity profile was assumed to be a linear function of ξ. In order to determine the density profile, it requires a further assumption that the term $2\theta\eta(\partial\psi/\partial\eta)$ be negligibly small. This term is the same order of magnitude as the other terms in Eq. (1) for low shock Mach numbers. Hence the results become increasingly inaccurate as $1/M_s{}^2 \rightarrow 1$ in the asymptotic regime. However, if an appropriate form for the density profile is assumed, then Eq. (1) reduces to a first-order ordinary linear differential equation which can be solved immediately for the particle velocity $\phi(\xi,\eta)$ without involving further approximations. Following Porzel[15] and Rae, we assumed the density profile to be of the form

$$\psi(\xi,\eta) = \psi(1,\eta)\xi^{q(\eta)} \quad (10)$$

The exponent $q(\eta)$ can be determined by substituting Eq. (10) into Eq. (4) and evaluating the mass integral, an algebraic equation for $q(\eta)$ is obtained. Solving for $q(\eta)$ one finds

$$q(\eta) = (j + 1)[\psi(1,\eta) - 1] \quad (11)$$

It should be noted that the density profile is completely specified for every local shock Mach number by Eqs. (9–11).

With the density profile known, its derivatives with respect to ξ and η can be readily evaluated. Substituting these into Eq. (1), one obtains the following differential equation for ϕ:

$$\frac{\partial\phi}{\partial\xi} + (q + j)\frac{\phi}{\xi} = q + \frac{2\theta\eta}{\psi(1,\eta)}\,[1 + (j + 1)\psi(1,\eta)\ln\xi]\,\frac{d\psi(1,\eta)}{d\eta} \quad (12)$$

Solving Eq. (12) subject to boundary condition that $\phi(0,\eta) = 0$, one obtains the particle velocity profile to be

$$\phi = \phi(1,\eta)\xi(1 - \Theta\ln\xi)$$

where

$$\Theta = \frac{-2\theta\eta}{\phi(1,\eta)\psi(1,\eta)}\cdot\frac{d\psi(1,\eta)}{d\eta} \quad (13)$$

Note that unlike the density profile, the particle velocity is not explicitly determined for any local shock Mach number since it contains the variable $\theta(\eta)$ which is as yet unknown. In Sakurai's analysis, the velocity is assumed to be linear, hence the term Θ is neglected. Under this approximation, the velocity profile is also completely specified for any local shock Mach number. The present exact form for the particle velocity profile [i.e., Eq. (13)] reduces to a linear function of ξ in the strong shock limit as $\eta \rightarrow 0$.

Substituting the density profile, the particle velocity profile and their derivatives with respect to ξ and η into the momentum equation [i.e., Eq. (2)] one obtains

$$f = -\int\left[-2\theta\eta\xi\left(\frac{d}{d\eta}\,\phi(1,\eta) - \frac{d}{d\eta}\,[\phi(1,\eta)\Theta]\ln\xi\right) + (\phi - \xi)\phi(1,\eta)(1 - \Theta - \Theta\ln\xi) + \theta\phi\right] \times \psi(1,\eta)\xi^{q(\eta)}d\xi + C(\eta) \quad (14)$$

Equation (14) can be integrated directly and to evaluate the constant of integration $C(\eta)$, the boundary condition at the shock front [i.e., $\xi = 1, f(\xi,\eta) = f(1,\eta)$] is used. After some algebraic manipulations, the pressure profile can be written as

$$f(\xi,\eta) = f(1,\eta) + f_2(\xi^{q+2} - 1) + f_3\{\xi^{q+2}[(q + 2)\ln\xi - 1] + 1\} + f_4\{2 - \xi^{q+2}[(q + 2)^2\ln^2\xi - 2(q + 2)\ln\xi + 2]\} \quad (15)$$

where $f(1,\eta)$ is the boundary condition at the shock front given by Eq. (8) and

$$f_2 = \frac{\psi(1,\eta)}{(q + 2)}\left[(1 - \Theta)\{\phi(1,\eta) - \phi^2(1,\eta)\} - \theta\left\{\phi(1,\eta) - 2\eta\,\frac{d\phi(1,\eta)}{d\eta}\right\}\right]$$

$$f_3 = \frac{\psi(1,\eta)}{(q + 2)^2}\left(\theta\left\{\Theta\phi(1,\eta) - 2\eta\,\frac{d}{d\eta}\,[\Theta\phi(1,\eta)]\right\} - \Theta\phi(1,\eta) - \Theta^2\phi^2(1,\eta) + 2\Theta\phi^2(1,\eta)\right)$$

$$f_4 = \Theta^2\phi^2(1,\eta)\psi(1,\eta)/(q + 2)^3$$

In the strong shock limit when $\eta \rightarrow 0$, the pressure profile simplifies to the following form

$$f(\xi) = f(1,0) + \frac{\psi(1,0)\phi(1,0)}{(q + 2)}\,(\xi^{q+2} - 1)[1 - \phi(1,0) - \theta(0)] \quad (16)$$

This relationship is identical to that obtained by Sakurai for the pressure profile. Summarizing one notes that in Sakurai's analysis, the profiles are always assumed to take on the simplified form valid only for strong shock waves. Hence in the asymptotic weak shock regime, Sakurai's approximate solution becomes quite inaccurate.

In the pressure profile given by Eq. (15), one notes that it requires both the value of $\theta(\eta)$ and its first derivative $d\theta(\eta)/d\eta$ to be known before it can be evaluated.

2.2 Energy Integral

To complete the solution, the functional relationship between θ and η must be determined. Substituting the density, particle velocity, and the pressure profiles [i.e., Eqs. (10, 13, and 15)] into the energy integral [i.e., Eq. (5)] and solving for

$d\theta(\eta)/d\eta$ one obtains the following equation:

$$\frac{d\theta}{d\eta} = -\frac{1}{2\eta}\left\{\theta + 1 - 2\phi(1,\eta) - \frac{(D_1 + 4\eta)}{\gamma + 1} - (\gamma - 1)(j + 1)\left[\phi(1,\eta) - \frac{(D_1 + 4\eta)^2}{4\theta y(\gamma + 1)}\right]\right\} + \frac{(D_1 + 4\eta)}{8\eta^2(\gamma + 1)}\left[\frac{(D_1 + 4\eta)\phi(1,\eta)}{\theta} - \frac{\phi(1,\eta)(\gamma + 1)}{\theta\psi(1,\eta)} + 2(\eta + 1) + (\gamma - 1)(j + 1)\frac{(\gamma + 1)}{2\theta}\phi^2(1,\eta)\right] + \frac{2\theta[2 + (\gamma - 1)(j + 1)]}{D_1 + 4\eta} \tag{17}$$

where $D_1 = \gamma(j + 3) + (j - 1)$.

In the Eq. (17), the relationship for the dependence of the shock radius y on the shock strength η is unknown. Hence an additional equation for $y(\eta)$ is required. This can be obtained directly from the definition of $y(\eta)$ itself. Differentiating Eq. 6 with respect to the shock radius R_s and noting the identity

$$R_s d/dR_s = \theta M_s d/dM_s = -2\theta\eta d/d\eta$$

one obtains

$$dy/d\eta = -(j + 1)y/2\theta\eta \tag{18}$$

Equations (17) and (18) constitute a pair of first-order ordinary differential equations for $\theta(\eta)$ and $y(\eta)$ which can be readily integrated by standard numerical methods.

To determine the shock trajectory, the identity

$$\dot{R}_s = dR_s/dt \tag{19}$$

is used. In terms of the dimensional parameters y and θ, Eq. (19) can be expressed in the following form:

$$\frac{c_0 t}{R_0} = -\frac{1}{2}\int_0^\eta \frac{y^{1/j+1}d\eta}{\theta\eta^{1/2}} \tag{20}$$

Hence as the solutions for $y(\eta)$ and $\theta(\eta)$ are being determined, the integral in Eq. (20) can simultaneously be carried out numerically yielding the shock trajectory.

2.3 Numerical Solutions

To determine $\theta(\eta)$ and $y(\eta)$, the pair of differential equation [i.e., Eqs. (17) and (18)] must be integrated numerically with the boundary conditions at $\eta = 0$

$$\theta(0) = \theta_0, \; y(0) = 0$$

However, one notes that both Eqs. (17) and (18) are singular at $\eta = 0$ since that boundary condition states that $y(0) = 0$. Hence the slopes $d\theta/d\eta = 0/0$ and $dy/d\eta = 0/0$ are indeterminate. Therefore to perform the numerical integration, it is necessary to evaluate both θ and y at some small finite value of η and proceed the integration from this point. To seek solutions in the neighborhood of $\eta = 0$, $\theta(\eta)$ and $y(\eta)$ are expanded in the following power series:

$$\theta(\eta) = \theta_0 + \theta_1\eta + \theta_2\eta^2 + \ldots$$
$$y(\eta) = y_1\eta + y_2\eta^2 + y_3\eta^3 + \ldots \tag{21}$$

Substituting the preceding perturbation series into Eqs. (17) and (18) and equating terms of similar powers in η, the coefficients θ_n and y_n can readily be determined. Since the algebraic expressions for these coefficients are rather lengthy, they are tabulated in the Appendix. It should be noted that the coefficient θ_0 was found to be

$$\theta_0 = -(j + 1)/2$$

which yields the time exponent $N = \frac{2}{5}, \frac{1}{2}$, and $\frac{2}{3}$ for spherical cylindrical and planar waves as in the classical self-similar solution.

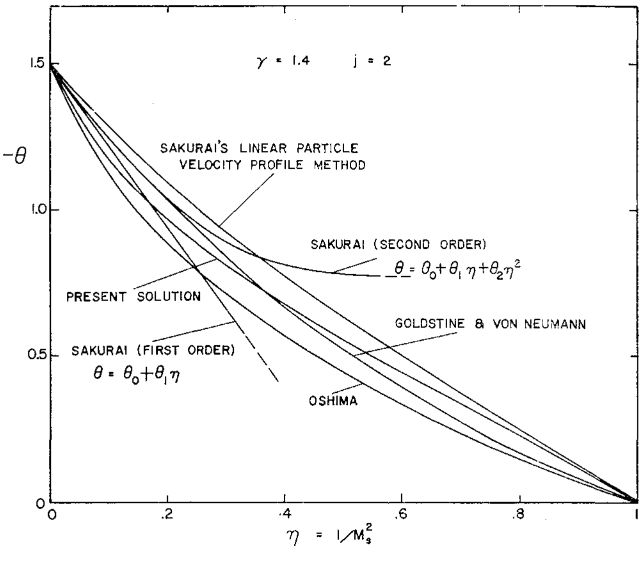

Fig. 1 The variation of the shock decay coefficient θ vs shock strength η for spherical blast waves, $\gamma = 1.4$.

For given geometry "j" and the specific heat ratio "γ," the perturbation coefficients θ_n and y_n are first determined. With $\theta(\eta^*)$ and $y(\eta^*)$ evaluated at some small value of $\eta = \eta^*$ where $\eta^* \ll 1$, Eqs. (17) and (18) are then integrated numerically using the Range-Kutta method. In the present work, we found that using the Range-Kutta method, the step size h used must be small as compared to η^* in the early phases of the integration, otherwise large errors are generated. It was found that using $h/\eta^* \leq 0.05$ gave good results.

For the shock trajectory, an analytical expression can be obtained for $\eta \ll 1$ using the perturbation series for $y(\eta)$. Substituting Eq. (21) into Eq. (20) and evaluating the integral one obtains

$$\frac{c_0 t}{R_0} = 2y_1^{1/j+1}\eta^{[(j+3)/2(j+1)]}\left[\frac{1}{j + 3} + \frac{T_1\eta}{3j + 5} + \frac{T_2\eta^2}{5j + 7} + \frac{T_3\eta^3}{7j + 9} + \ldots\right] \tag{22}$$

where

$$T_1 = y_2(j + 2)/[y_1(j + 1)]$$
$$T_2 = (2j + 3)\{y_3/y_1 - jy_2^2/[2(j + 1)y_1^2]\}/(j + 1)$$
$$T_3 = (3j + 4)\{y_4/y_1 - jy_2y_3/[y_1^2(j + 1)] + j(2j + 1)y_2^3/[6y_1^3(j + 1)^2]\}/(j + 1)$$

3.0 Results and Discussions

The solutions for $\theta(\eta)$ and $Rs/R_0(\eta)$ for the case of spherical blast wave in air (i.e., $j = 2$, $\gamma = 1.4$) are shown in Figs. 1 and 2, respectively. Also plotted for comparison are the results from the exact solution of Goldstine and von Neuman,[13] the perturbation solution of Sakurai,[10] the quasi-similar solution of Oshima as computed by Lewis,[17] and the approximate linear velocity profile solution of Sakurai.[16] As indicated in Fig. 2, all the approximate analytical solutions agree quite well with the exact solution in the strong and moderate shock strength regime (i.e., $2 \leq M_s \leq \infty$). For the asymptotic weak shock regime (i.e., $M_s < 2$) the present solution gave remarkably accurate results. This is perhaps due to the fact that the present method satisfies the boundary conditions at the shock front and the conservation of total mass and energy at all times. Hence the solution is accurate as long as the density distribution behind the blast wave can ade-

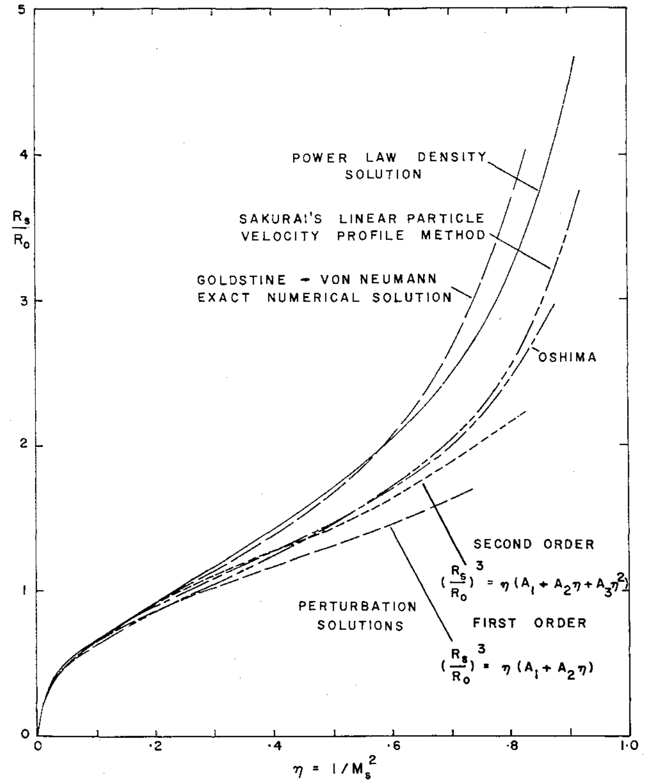

Fig. 2 The variation of shock strength η vs shock radius Rs/R_0 for spherical blast waves, $\gamma = 1.4$.

quately be described by a simple power law. For blast waves the power-law form for the density profile is quite valid except in the acoustic limit when $M_s \to 1$ and the density becomes virtually constant behind the shock. However, even in the acoustic limit when the profiles are poorly described by the present method, the fact that the integrated quantities such as total mass and energy are conserved should not affect the accuracy of the shock trajectory.

The pressure, density, and the particle velocity profiles for various shock Mach numbers are shown in Figs. 3–5, respectively. Of particular interest are the particle velocity profiles shown in Fig. 5 where the inclusion of the second term in Eq. 13 results in a region behind the blast wave where the velocity is negative (i.e. directed towards the center of symmetry) in the moderate and weak shock regime. The existence of such a negative velocity region corresponds to the results obtained by exact numerical methods as reported in Sedov's[9] monograph. From Fig. 3, one notes that the present pressure pro-

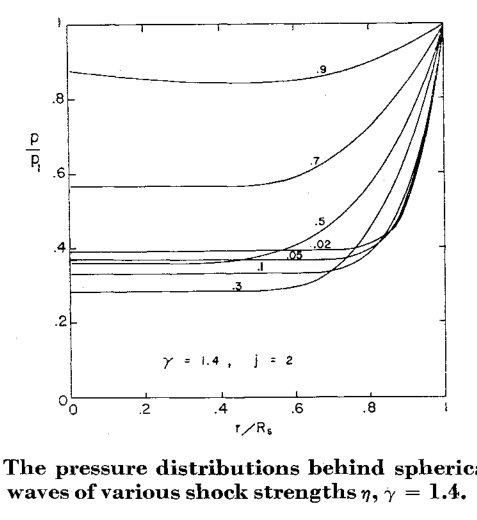

Fig. 3 The pressure distributions behind spherical blast waves of various shock strengths η, $\gamma = 1.4$.

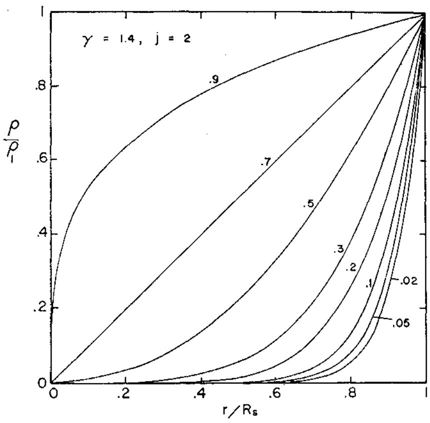

Fig. 4 The density distributions behind spherical blast waves of various shock strengths η, $\gamma = 1.4$.

files for very weak shocks tends to approach the N-wave form for spherical acoustic waves.

It should be noted that although the present method conserves the total energy enclosed by the blast wave at any instant of time, the energy distribution is not exact. The differential equation of energy conservation [i.e., Eq. (3)] will not be satisfied upon substitution of the density, particle velocity, and the pressure profiles [i.e., Eqs. (10, 13, and 15)] into it. The error in the energy profile is a direct consequence of the approximation involved in the assumption of the power-law density profile. For the case of strong spherical blast (i.e., $\eta = 0$) in a medium with $\gamma = 7$, the assumed power law density profile is exact and the energy equation [i.e., Eq. (3)] is satisfied.

Appendix

The coefficients θ_n's and y_n's of the perturbation expressions given in Eq. (21) are as follows:

$$\theta_0 = -(j + 1)/2$$

$$y_1 = -E_1/(C_1 + A_1\theta_0)$$

$$\theta_1 = \theta_0^2[A_2 + B_1\theta_0 + C_2/\theta_0 + E_2/(y_1\theta_0)]/C_1$$

$$y_2 = -y_1\theta_1/\theta_0$$

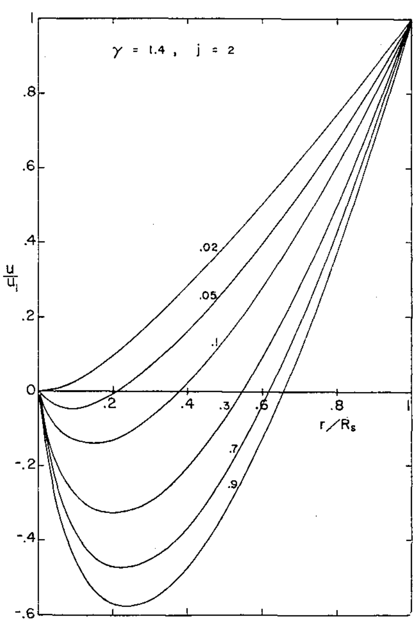

Fig. 5 The particle velocity distributions behind spherical blast waves of various shock strengths η, $\gamma = 1.4$.

$$\theta_2 = \theta_0^2[A_3 + (B_1 - 1)\theta_1 + B_2\theta_0 + (C_1\theta_1^2/\theta_0^2 - C_2\theta_1/\theta_0 + C_3)/\theta_0 + E_3/(y_1\theta_0)]/[C_1 + E_1/(2y_1)]$$

$$y_3 = y_1[\theta_1^2/\theta_0^2 - \theta_2/(2\theta_0)]$$

$$\theta_3 = \theta_0^2((B_1 - 2)\theta_2 + B_2\theta_1 + B_3\theta_0 + \{C_1[2\theta_1\theta_2/\theta_0^2 - \theta_1^3/\theta_0^3] + C_2(\theta_1^2/\theta_0^2 - \theta_2/\theta_0) - C_3\theta_1/\theta_0 + C_4\}/\theta_0 + \{E_1\theta_1\theta_2/(3\theta_0^2) - E_2\theta_2/(2\theta_0)\}/(y_1\theta_0))/\{C_1 + 2E_1/(3y_1)\}$$

$$y_4 = y_1\{7\theta_1\theta_2/(2\theta_0^2) - 3\theta_1^3/\theta_0^3 - \theta_3/\theta_0\}/3$$

where the A's, B's, etc. are given by

$$A_1 = D_1/[4(\gamma + 1)]$$

$$A_2 = -\tfrac{1}{2} + [2 + (\gamma - 1)(j + 1)]/(\gamma + 1) + (3D_1 + 4)/[4(\gamma + 1)]$$

$$A_3 = [1 - (\gamma - 1)(j + 1)]/(\gamma + 1)$$

$$B_1 = -\tfrac{1}{2}$$

$$B_2 = 2[2 + (\gamma - 1)(j + 1)]/D_1$$

$$B_3 = -8[2 + (\gamma - 1)(j + 1)]/D_1^2$$

$$C_1 = D_1[D_1 + j(\gamma + 1)]/[4(\gamma + 1)^2]$$

$$C_2 = \{-D_1^2 + D_1[6 - (\gamma - 1)(2j + 1)] + 4j(\gamma - 1)\}/[4(\gamma + 1)^2]$$

$$C_3 = \{D_1[-6 + (\gamma - 1)(j + 1)] + 4[2 - (\gamma - 1)(2j + 1)]\}/[4(\gamma + 1)^2]$$

$$C_4 = -[2 - (\gamma - 1)(j + 1)]/(\gamma + 1)^2$$

$$E_1 = -(\gamma - 1)(j + 1)D_1^2/[8(\gamma + 1)]$$

$$E_2 = -(\gamma - 1)(j + 1)D_1/(\gamma + 1)$$

$$E_3 = -2(\gamma - 1)(j + 1)/(\gamma + 1)$$

and

$$D_1 = \gamma(j + 3) + j - 1$$

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